EFFECT OF A MAGNETIC FIELD ON THE FLOW IN THE VICINITY OF THE STAGNATION POINT OF A BLUNT BODY WITH ABLATION OF A PROTECTIVE LAYER

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During hypersonic flow around a blunt-nosed body, the gas which passes through the bow shock is heated to high temperatures, where dissociation, ionization, and inverse phenomena (recombination) take place in the gas. If an ionized gas moves in a magnetic field, the ponderomotive force which is set up changes the nature of its motion close to the stagnation point, decreasing the frictional stress and heat transfer at the "wall" (at the contact surface of the gas and the body about which the gas flows). In this case, the intense heat fluxes from the strongly heated gas to the body about which the gas flows cause phase changes in the surface of the body (melting, sublimation, etc.). These processes, in turn, affect the flow in the vicinity of the stagnation point due to realization of the heat of phase transition, the conduction of heat from the entrained mass, and the diffusion of evaporating material into the boundary layer. References [1, 2] are devoted to a study of the joint influence of the magnetogasdynamic and ablation effects. The magnetogasdynamic layers and the wall profile of the external velocity ("flow around wedges") are discussed in [1], and special cases of such boundary layers—flow close to the stagnation line (the two-dimensional case) and close to the stagnation point (the axisymmetric case) of a blunt body are considered in [2]. Melting and evaporation are taken into account by setting the longitudinal and the transverse velocity components at the wall not equal to zero—the first taking into account the flow of the molten material and the second pyrolysis of the vapor of the surface material into the gaseous boundary layer. However, the values of these components, also the enthalpy on the wall h_w (in [1, 2], $h_w \approx 0$), are not known beforehand and must be determined from the boundary conditions at the wall which express the mass and heat balances. The general formulation of the problem given in the gasdynamics case by G. A. Triskii in [3, 4], and elsewhere includes a consideration of the boundary-layer equations in the gas, the boundary-layer equations in the melted zone, and the heat conductivity equations in the solid with boundary conditions at the outer edge of the boundary layer, on the gas-molten zone interface, on the molten zone-solid interface, and inside the solid. This approach to the problem can also be utilized in the magnetogasdynamic case, as it is in this article with certain simplifying assumptions as compared with [3, 4]. In this sense, the present article is an extension of the results of [3, 4] to the field of magnetogasdynamics.

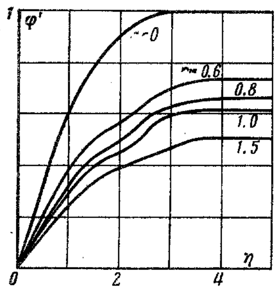

Fig. 1

1. We shall consider the plane and axisymmetric flow of a viscous, heat-conducting, and electrically conducting gas about a semi-infinite blunt body (or more precisely, a mixture of interacting gases; in the case of sublimation, vapors of the substance composing the surface of the body are added to the gases).

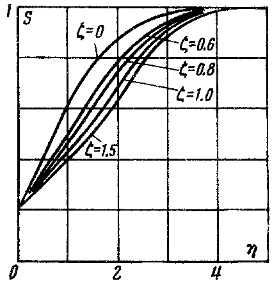

Fig. 2

A homogeneous magnetic field perpendicular to the surface of the body is applied; there is no external electric field; then it can be shown [5] that no induced electric field is set up and the gas has no excess charge. The gas behind the shock wave is considered to be quite dense and the applied magnetic field to be weak, so that the Hall effect can be neglected, and the electrical conductivity can be considered a scalar (in addition, we shall consider the electrical conductivity of the gas and molten mass to be constants). Making use of the fact that the viscous boundary layer is thin and the electrical conductivity due to thermal ionization insignificant (the magnetic Reynolds number R_m is small), we can neglect the effect of the induced magnetic field as compared with the applied field. Then, the magnetic induction **B** turns out to be constant and equal to the induction of the applied field $\mathbf{B}_0$, and the ponderomotive force retains only the longitudinal component

$$F_x = -\sigma_i B_0^2 u_i \qquad (i = 1,2) .$$

Here x is the coordinate axis parallel to the surface of the body; u is the projection of the velocity on this axis; σ is the electrical conductivity, the subscript 1 is associated with gas, and 2 with the molten zone; here and henceforth, the quantities associated with the gas will be understood to be the total for the mixture of gases.

We shall make the same assumptions in regard to the gasdynamic quantities as those in [3]. We shall assume that the shape of the body changes little in the process of phase transformation; then the distribution of pressures above the surface of the body will be the same as before melting or sublimation. In [3], the authors have neglected the terms in the energy equation of the order of M_∞^2 (where M_∞ is the Mach number behind the shock wave, $M_\infty^2 \ll 1$ in strong shocks).

These terms are the work of the pressure forces and and the viscous heat losses. We note that the term in the energy equation which takes account of joule losses is of the order

$$N_m^2 M_\infty^2, \qquad N_m^2 = \frac{\sigma_i B_0^2 l}{\rho_i u_\infty}.$$

Here N_m is the magnetic parameter; l, u_∞, and ρ_i are the scales of length, velocity, and density, respectively. With magnetic parameters $N_m \sim 1$, the joule losses are of the same order as the work of the forces of pressure and the viscous heat losses, and are rejected along with them. Finally, as in [4], we neglect thermal diffusion and consider all diffusion coefficients equal.

With these assumptions, the equations of the boundary layer in the gas and in the molten zone differ from the corresponding equations of the gasdynamic approximation [3, 4] only by a term which takes account of the ponderomotive force in the equation of motion. As in the case without a magnetic field, a stationary self-similar mode is possible with an arbitrary temperature dependence of the properties of the solid, liquid, and gas, if these relationships are expressed by known differential functions. For further simplification of the problem we shall consider the thermophysical properties of the solid and the liquid to be constant, and assume in the case of the gas

$$\frac{h}{h_\infty} = \frac{\rho_\infty}{\rho}, \qquad \frac{\rho\mu}{\rho_\infty \mu_\infty} = \text{const},$$
$$P = \frac{\mu c_p}{\lambda} = \text{const}, \qquad L = \frac{\rho c_p D}{\lambda} = 1.$$

Here h is the enthalpy, ρ the density, μ the dynamic viscosity, c_p the specific heat at constant pressure, λ the heat conductivity, P the Prandtl number, D the diffusion coefficient, L the Lewis number, and the subscript ∞ is associated with the potential flow behind the shock. Finally, we shall consider only the limiting thermochemical state—the "frozen" boundary layer (the rate of recombination $w \approx 0$). Under these conditions, the diffusion equation coincides with the energy equation [6] and can be integrated separately after the velocity distribution in the boundary layer is determined. Having this in mind, we shall not write the diffusion equation or the boundary conditions for concentrations of the components of the mixture.

Now, the stationary self-similar mode of flow in the vicinity of the stagnation point is written by means of the following dimensionless equations:

$$\varphi''' + n\varphi\varphi'' = \varphi'^2 - S + N_m^2\varphi' S,$$
$$S'' + nP\varphi s' = 0, \qquad (1.1)$$

$$\varphi_1''' + n\varphi_1\varphi_1'' = \varphi_1'^2 + N_{m1}^2\varphi_1',$$
$$\theta_1'' + nP\varphi_1\theta_1' = 0. \qquad (1.2)$$

The dimensionless quantities included in (1.1) and (1.2) are connected with the corresponding dimensional quantities by the following equations:

for the gas,

$$u = \beta x\varphi'(\eta), \qquad v = -\frac{\rho_\infty}{\rho}\sqrt{\beta\nu_\infty}\left[n\varphi(\eta) + \alpha\frac{\rho}{\rho_\infty}\right],$$

$$h = h_\infty S(\eta), \quad p = p_\infty - \frac{\rho_\infty \beta^2}{2}x^2, \quad \alpha = \frac{a^*}{\sqrt{\beta\nu_\infty}},$$

$$\beta = \left(\frac{\partial u_\infty}{\partial x}\right)_{\substack{x=0\\ N_m=0}}, \qquad \int_0^\eta \frac{\rho_\infty}{\rho}d\eta = \left(\frac{\beta}{\nu_\infty}\right)^{1/2}(y + a^*t),$$

$$\nu_\infty = \frac{\mu_\infty}{\rho_\infty}, \quad N_m^2 = \frac{\sigma B_0^2}{\rho_\infty\beta} \quad (n = 1, 2);$$

for the molten film,

$$u_1 = \beta_1 x\varphi_1'(\eta_1), \quad v_1 = -\sqrt{\beta_1\nu_1}\,[n\varphi_1(\eta_1) + \alpha_1],$$

$$T_1 = T_*\theta_1(\eta_1), \quad \nu_1 = \frac{\mu_1}{\rho_1},$$

$$p = p_{00} - \frac{\rho_1\beta_1^2}{2}x^2, \quad \alpha_1 = \frac{a^*}{\sqrt{\beta_1\nu_1}},$$

$$\eta_1 = \left(\frac{\beta_1}{\nu_1}\right)^{1/2}(y + a^*t), \quad N_{m1}^2 = \frac{\sigma_1 B_0^2}{\rho_1\beta_1}.$$

Here n = 1 corresponds to plane and n = 2 to axisymmetric flows, the quantities without any subscript to the gas, with the subscript 1 to the liquid (molten mass); x is the distance from the stagnation point (line) along the surface of the body; y is the distance from the surface of the body along the normal to it; u and v are the projections of the velocity vector on the x and y-axes, respectively; t is time; T absolute temperature, p pressure, a^* the rate of displacement of the melting front normal to the surface of the body, T^* is the melting point, ν the kinematic viscosity, p_{00} the pressure at the stagnation point; η, η_1 are dimensionless coordinates; and α, α_1 are the dimensionless velocities of the interfaces.

Within the framework of these assumptions, the boundary conditions for system (1.2) will be the same as in the gasdynamics formulation. The only exception is the condition for the longitudinal component of the velocity on the outer boundary of the boundary layer. In accordance with [1], we obtain from the equation of motion when $y \to \infty$ (in the dimensionless form from the first equation of (1.1) when $\eta \to \infty$),

$$\lim_{y\to\infty} u = u_\infty = u_*[-{}^1/_2 N_m^2 + ({}^1/_4 N_m^4 + 1)^{1/2}],$$
$$\lim_{\eta\to\infty} \varphi' = -{}^1/_2 N_m^2 + ({}^1/_4 N_m^4 + 1)^{1/2}. \qquad (1.3)$$

Here $u_* = \beta x$ is the velocity at the outer edge of the boundary layer in the absence of a magnetic field. We note that the authors of [2] consider that allowing for changes in the velocity u_∞ is superfluous when there is a field and set $\varphi'(\infty) = 1$.

The remaining boundary conditions do not change form. We shall write them for the case of melting (only one example cited in the article is associated with evaporation; the case is not discussed in detail). In the equalities written below, $\eta = 0$ corresponds to the surface of the body $\eta = -\eta_*$ is the melting front:

$$S \to 1 \quad \text{as} \quad \eta \to \infty, \tag{1.4}$$

$$\varphi(0) = \varphi_1(0) = 0, \qquad \varphi'(0) = \left(\frac{\rho_\infty}{\rho_1}\right)^{1/2} \varphi_1'(0),$$

$$B\varphi''(0) = \varphi_1''(0) \qquad S(0) = m\theta_1(0),$$

$$qS'(0) = m\theta_1'(0), \tag{1.5}$$

$$\varphi_1'(-\eta_*) = 0, \qquad \theta_1(-\eta_*) = 1, \tag{1.6}$$

$$r_2 n\varphi_1(-\eta_*) = -\alpha_1,$$

$$\theta_1'(-\eta_*) = \frac{\alpha_1}{l_2}[\Delta + P_2(1-k)]. \tag{1.7}$$

Here, the magnetic parameters determined above are added to the notation of [3],

$$P = \frac{\mu_\infty c_p}{\lambda_\infty}, \quad P_1 = \frac{\mu_1 c_1}{\lambda_1}, \quad P_2 = \frac{\mu_1 c_2}{\lambda_2}, \quad r_2 = \frac{\rho_1}{\rho_2},$$

$$m = \frac{c_{pw} T_*}{h_\infty}, \quad q = l_1\left(\frac{\nu_\infty}{\nu_1}\right)^{-1/2}\left(\frac{\rho_\infty}{\rho_1}\right)^{-1/4}, \quad l_1 = \frac{\lambda_\infty}{\lambda_1},$$

$$B = \left(\frac{\rho_\infty}{\rho_1}\right)^{1/4}\left(\frac{\nu_\infty}{\nu_1}\right)^{1/2}, \quad l_2 = \frac{\lambda_1}{\lambda_2},$$

$$k = \frac{T_{20}}{T_*}, \quad \Delta = \frac{\rho_2 \nu_1 \delta}{\lambda_2 T_*}, \quad \eta_* = b\left(\frac{\beta_1}{\nu_1}\right)^{1/2}.$$

In the last equalities, in addition to the quantities defined previously, we have: T_{20} the temperature in the bulk of the solid (with y, $\eta \to -\infty$), b the thickness of the molten layer, and δ the specific heat of melting. In the formulation of the boundary-layer problem (1.1)-(1.7), the heat conductivity equation for the solid was integrated beforehand, and the result,

$$\theta_2 = \frac{T_2}{T_*} = (1-k)\exp(\alpha_1 P_2(\eta_* + \eta)) + k,$$

where T_2 is the temperature in the solid, substituted into the last of boundary conditions (1.16).

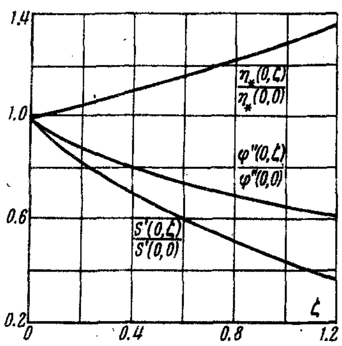

Fig. 3

Thus, we have established the boundary-value problem for a system of nonlinear tenth-order ordinary differential equations (1.1), (1.2) with ten boundary conditions (1.3)–(1.6). Two relations (1.7), the equations of conservation of mass and heat on the melting surface, yield two transcendental equations in the unknown parameters α_1, the dimensionless velocity of the melting front, and η_*, the dimensionless thickness of the molten layer.

2. In the first instance, we shall consider the mode in which the electrical conductivity of the oncoming gas is negligibly small and the magnetic field affects only the flow of electrically conducting molten mass. In this case, $N_m = 0$, and we can make use of the solution for a gas in the gasdynamics case. From this solution, we obtain the values of the dimensionless frictional stresses $\varphi_1''(0)$ and heat transfer $\varphi_1'(0)$ at the gas-molten zone interface in two arguments—the dimensionless temperature $\theta_1(0)$ and the dimensionless longitudinal velocity $\varphi_1'(0)$.

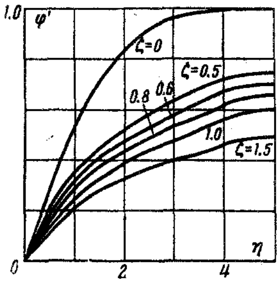

Fig. 4

We shall seek the solution of the first of Eqs. (1.2) with its associated boundary conditions from (1.5), (1.6) in the form of a segment of the MacLaurin series for the unknown function φ_1'. Making use of the fact that the molten layer is thin, we restrict ourselves to the terms $O(\eta_3)$,

$$\varphi_1' = \sum_{k=0}^{3} \frac{\varphi_1^{(k+1)}}{k!}\eta^k + O(\eta^4). \tag{2.1}$$

The solution of the second equation of (1.2) with known φ_1' is reduced to a quadrature. We write $\varphi_1'(0) = \varepsilon_1$, $\varphi_1''(0) = a_1$, $Nm_1^2 = \zeta_1$, and determine the coefficients in (2.1) from the differential equation (1.2) when n = 1 (the two-dimensional case),

$$\varphi_1'(\eta) = \varepsilon_1 + a_1\eta + {}^1/_2(\varepsilon_1^2 + \zeta_1\varepsilon_1 - 1)\eta^2 +$$

$$+ {}^1/_6(\varepsilon_1 + \zeta_1)a_1\eta^3 + O(\eta^4). \tag{2.2}$$

Satisfying the first of conditions (1.6), we obtain the relationship between the dimensionless thickness of the layer η_* and parameters ε_1 and a_1, which we shall solve, for convenience, for ε_1,

$$\varepsilon_1 = a_1\eta_* + {}^1/_2\eta_*^2 - {}^1/_3 a_1\zeta\eta_*^3 + O(\eta_*^4). \tag{2.3}$$

Making use of (2.2) and (2.3), we can determine

$$\varphi_1(-\eta_*) = \int_0^{-\eta_*}\varphi_1'(\eta)\,d\eta$$

and from the first relation of (1.7), the dimensionless velocity of the melting front,

$$\alpha_1 = r_2({}^1/_2 a_1\eta_*^2 + {}^1/_6\eta_*^3 - {}^1/_6 a_1\zeta\eta_*^4) + O(\eta_*^5). \tag{2.4}$$

In addition to the velocity of the melting front and the thickness of the molten layer, we are also interested in the protective effect of the molten film, which is expressed in the removal of the heat of this film. Taking (2.2) and (2.3) into consideration, we obtain from the second equation of (1.2)

$$\vartheta = \frac{\theta_1'(-\eta_*)}{\theta_1'(0)} = \exp\left(-P_1 \int_0^{-\eta_*} \varphi_1 d\eta_1\right) =$$
$$= 1 - P_1 (^1/_3 a_1 \eta_*^3 + {}^5/_{24} \eta_*^4) + O(\eta_*^5). \qquad (2.5)$$

Substituting (2.4) and (2.5) into the second relation of (1.7), we obtain an algebraic equation in η_*

$$\frac{l_2 \theta_1'(0)}{r_2 [\Delta + P_2(1-k)]} = \frac{^1/_2 a_1 \eta_*^2 + {}^1/_6 \eta_*^3 - {}^1/_6 a_1 \zeta_1 \eta_*^4}{1 - P_1(^1/_3 a_1 \eta_*^3 + {}^5/_{24} \eta_*^4)}. \quad (2.6)$$

The quantity $\theta_1'(0)$, the heat flux from the gas to the molten film, can be considered as independent of the magnetic parameter ζ_1, as the gas is not electrically conductive; then the left side of (2.6) does not depend on ζ_1 and may be of the same form when $\zeta_1 = 0$. We shall call the root of the equation changed in this manner η_{10}, then Eq. (2.6) can be rewritten in the form

$$\frac{^1/_2 a_1 \eta_{10}^2 + {}^1/_6 \eta_{10}^3}{1 - P_1(^1/_3 a_1 \eta_{10}^3 + {}^5/_{24} \eta_{10}^4)} = \frac{^1/_2 a_1 \eta_*^2 + {}^1/_6 \eta_*^3 - {}^1/_6 a_1 \zeta \eta_*^4}{1 - P_1(a_1 \eta_*^3 + {}^5/_{24} \eta_*^4)}. \quad (2.7)$$

We can now draw some conclusions concerning the effect of the magnetic field on the values of the ablation parameters—the thickness of the layer, the velocity of the melting front, and the protective effect. It is convenient to express these quantities in terms of the dimensionless thickness of the layer in the absence of a magnetic field η_{10}. We shall begin by solving Eq. (2.7). We solve it by the method of tangents, beginning with the zero-th approximation $\eta_*^{(0)} = \eta_{10}$ and limiting ourselves to the first approximation. We obtain as a result

$$\eta_* = \eta_{10} + \frac{\zeta_1 \eta_{10}^3}{6} - \frac{\zeta_1 \eta_{10}^4}{12 a_1} + O(\eta_1^5).$$

Substituting the last result in (2.4) and (2.5) and relating the values of the parameters in the presence of a magnetic field to their values in the gasdynamics case (we shall give these values the subscript 0), we derive the following approximate formulas:

$$\eta_* / \eta_{10} = 1 + {}^1/_6 \zeta_1 \eta_{10}^2 + O(\eta_{10}^3),$$
$$a_1 / a_{10} = 1 + {}^1/_{36} (a_1^2 \zeta_1 + 3\zeta_1 - 8 a_1^2 \zeta_1^2) \eta_{10}^4 + O(\eta_{10}^5)$$
$$\vartheta / \vartheta_0 = 1 + {}^1/_6 a_1 \zeta_1 \eta_{10}^5 + O(\eta_{10}^6). \qquad (2.8)$$

It can be seen from (2.8) that only the thickness of the molten layer undergoes more or less definite changes under the action of a magnetic field, while the velocity of the melting front and the protective effect vary practically not at all. The same result was obtained in Boynton's experiments [7].

3. Changes in the mode of melting can be expected only when the oncoming gas is electrically conductive. Let us consider the system of equations for the gaseous boundary layer (1.1) in this case. The boundary conditions for this system are (1.3), (1.4), the first, second, and fourth from (1.5). For typical protective layer materials, the quantity $(\rho_\infty/\rho_1)^{1/2} \ll 1$, so that the value of the longitudinal velocity component $\varphi'(0) = \varepsilon$ is small.

We shall first solve system (1.1) for the condition $\varphi'(0) = 0$; we shall take the effect of the flow of molten material ($\varepsilon \neq 0$) into consideration later.

With a view to finding the solution of system (1.1) in the form of a series in a small parameter, we choose this parameter on the basis of the following considerations.

We rewrite (1.3) in the form

$$\varphi'(\infty) = -{}^1/_2 N_m^2 + (^1/_4 N_m^4 + 1)^{1/2} = 1 - z. \quad (3.1)$$

Here z is the deviation of the velocity at the outer edge of the boundary layer from its value in the absence of a magnetic field. It is convenient to choose the quantity z as the small parameter. Indeed, it can be seen from (3.1) that when N_m^2 is changed from 0 to ∞, the magnitude of z changes from 0 to 1 and remains small even for large values of N_m^2 ($N_m^2 = 2.5$, $z = 0.5$; $N_m^2 = 5$, $z = 0.65$). Thus, the first approximations will be sufficient for considering cases of practical interest. The quantity N_m^2 in system (1.1) and the boundary condition (1.3) is expressed in terms of z in the following manner:

$$N_m^2 = \frac{z^2 + 2z}{1 - z} = 2z + 3 \sum_{n=2}^{\infty} z^n. \qquad (3.2)$$

We seek the functions φ and S in the form of series in powers of z,

$$\varphi = \sum_{n=0}^{\infty} \varphi_n z^n, \qquad S = \sum_{n=0}^{\infty} S_n z^n. \qquad (3.3)$$

Assuming that series (3.3) and the series obtained from them by formal differentiation (three and two times, respectively) are uniformly convergent, we substitute series (3.3) and the series for the derivatives into (1.1). Equating the coefficients of like powers of z, we obtain an infinite number of systems of fifth-order equations. We obtain the zero approximation by setting N_m^2 equal to zero in (1.1).

The system of equations of the linear approximation in z is of the form

$$\varphi_1''' + \varphi^\circ \varphi_1'' - 2\varphi^{\circ\prime} \varphi_1' +$$
$$+ \varphi^{\circ\prime\prime} \varphi_1 = 2\varphi^{\circ\prime} S', \quad S_1'' + \varphi^\circ S_1' = -\varphi_1 S^{\circ\prime}. \quad (3.4)$$

The boundary conditions of the zero-th approximation system will be the same as for the basic system (1.1), except that condition (3.1) will take the form

$$\varphi'(\infty) = 1. \qquad (3.5)$$

The boundary conditions for the first-approximation system are written in the form

$$\varphi_1(0) = \varphi_1'(0) = S_1(0) =$$
$$= S_1(\infty) = 0, \quad \varphi_1'(\infty) = -1. \qquad (3.6)$$

The boundary conditions for the following approximations will be completely homogeneous.

A solution of the zero-th approximation system was obtained by Cohen and Reshotko [8]. The coefficients of system (3.4) with S(0) =

= 0.2 were taken from this solution. The boundary-value problem (3.4), (3.6) was solved by the well-known method of reduction to the Cauchy problem. The Adams method was employed in numerical integration of (3.4) and the initial points were determined by expansion of the solution in a MacLaurin series about the neighborhood $\eta = 0$. The computations were carried out manually on a semiautomatic desk calculator. The results of the computations are presented in Figs. 1–3. In Figs. 1 and 2, profiles of the dimensionless longitudinal velocity $\varphi'(\eta)$ are given for values of $\zeta = 0$, 0.6, 0.8, 1.0, and 1.5 and the dimensionless enthalpy $S(\eta)$ in the linear approximation in z. Curves $\varphi'(\eta)$ and $S(\eta)$ were constructed for different values of the magnetic parameter $\zeta = N_m^2$. It can be seen from the curves in Figs. 1 and 2 that the tangential stress $\varphi''(0)$ and the heat flux $S'(0)$ at the wall drop with increasing magnetic field. The variation of these parameters as a function of the magnetic parameter ζ is shown in Fig. 3.

One may conclude from formulas (2.5) and (1.7) that in the case of materials with a low Prandtl number of the liquid phase P_1, the protective effect is small (2.5) and the velocity of the melting front is proportional to the heat flux on the gas-molten zone interface $S'(0)$. Thus, the curve in Fig. 3, which shows the variation in $S'(0)$ as a function of ζ, simultaneously shows the variation in a as a function of the same parameter. The third curve of Fig. 3 shows the variation in the dimensionless thickness of the molten layer as a function of the magnetic parameter ζ. This curve was obtained from (2.6); the parameters in the left side of (2.6) were calculated on the basis of the thermophysical properties of cobalt. Thus, the interaction of the magnetic field with the oncoming electrically conductive gas leads to retardation of melting and thickening of the molten layer.

If we replace the first of conditions (1.5) by the condition $\varphi(0) = \varphi_W \neq 0$, then, with the reservations stated at the beginning of this article, system (1.1) will describe a laminar boundary layer in a gas with evaporation (blowing) from the wall. System (1.1) with the new boundary condition can be solved in precisely the same way as before, with $\varphi_W = 0$. The zero-th approximation was taken from the work of Beckwith [9] with $S(0) = 0.5$, $\varphi_W = -0.5$. The results of the calculations are presented in Figs. 4–6; Figs. 4 and 5 show families of profiles of the dimensionless velocity and enthalpy, Fig. 6 the variation in the tangential stress and the heat transfer on the wall as a function of the magnetic parameter (the notation in Fig. 6 is the same as in Fig. 3). It was found, in full agreement with references [1, 2] and other work, that in the presence of evaporation (injection), the effect of the magnetic field is more pronounced than when $\varphi_W = 0$. The curves of Figs. 4–6 were constructed, as before, in the linear approximation in the parameter z.

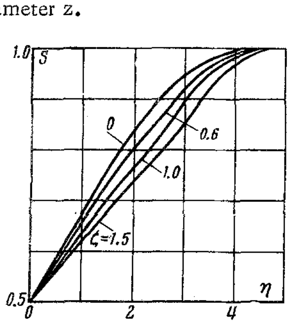

Fig. 5

4. Previously, in the boundary conditions for system (1.1), we set $\varphi'(0) = 0$, thus solving the problem for the boundary layer in gas as if a solid wall were present, without taking the flow of the molten mass into consideration. Let us turn to solving system (1.1) with the boundary condition $\varphi'(0) = \varepsilon \neq 0$ restored. We introduce the notation

$$\varphi''(0) = a(\varepsilon, S_w, P), \qquad S_w = S(0).$$

The values of the frictional stress at the wall $a(\varepsilon, S_W, P)$ can be found if we make use, as in [3], of the expansion of the solution in series in ε; at the same time, we take the boundary condition for the zero-th approximation for the function S_0 in the form

$$S_0(0) = t_0 = S_w - \frac{\varepsilon S_0'}{a(0, t_0, P)}.$$

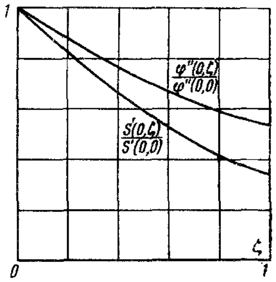

Fig. 6

After simple computations, we derive a formula which exactly coincides with the analogous formula in the gasdynamics case [3]

$$a(\varepsilon, S_w, P) = a(0, t_0, P) - \frac{\varepsilon S_w}{a(0, t_0, P)} + O(\varepsilon^2). \quad (4.1)$$

In order to determine the heat transfer when $\varepsilon \neq 0$, we make use of the formula

$$S'(0) = \frac{1 - S_w}{\omega(\infty, P, \varepsilon, S_w)}$$

$$\omega(\eta, P, \varepsilon, S_w) = \int_0^\eta \exp\left(-nP\int_0^{\eta_1} \varphi(\eta_2)\, d\eta_2\right) d\eta_1. \quad (4.2)$$

Here the integral is computed as in reference [3], with the aid of asymptotic integration as $\eta \to \infty$. Repeating the conclusion drawn in [3], we obtain

$$\omega(\infty, P, \varepsilon, S_w) = \frac{1}{3}\left(\frac{6}{anP}\right)^{1/3} \Gamma(1/3)\Big\{1 + \frac{S_w - \varepsilon^2 - \zeta\varepsilon S_w}{36}\left(\frac{6}{a}\right)^{4/3} \frac{\Gamma(2/3)}{\Gamma(1/3)}(nP)^{-1/3} +$$
$$+ \left[\frac{(S_w - \varepsilon^2 - \zeta\varepsilon S_w)^2}{16a^2} - \frac{(2-n)\varepsilon a - S'(0) + \zeta(aS_w + \varepsilon S'(0))}{20a} - \frac{nP\varepsilon}{2}\right]\left(\frac{6}{a}\right)^{2/3} \frac{\Gamma(1)}{\Gamma(1/3)}(nP)^{-2/3} + \left[\frac{35(S_w - \varepsilon^2 - \zeta\varepsilon S_w)^3}{648a^4} +\right.$$
$$+ \frac{(27n + 10)(\varepsilon + \zeta S_w)(\varepsilon^2 - S_w + \zeta\varepsilon S_w) - 2(2-n)a^2 - 4\zeta a S'(0)}{90a^2} +$$
$$\left. + \frac{7(S_w - \varepsilon^2 - \zeta\varepsilon S_w)[S'(0) - \zeta(aS_w + \varepsilon S'(0)]}{90a^3}\right]\frac{1}{nP} + \ldots\Big\}. \quad (4.3)$$

Here $\Gamma(x)$ is the Euler gamma function, n = 1 for plane and n = 2 for axisymmetric flows. When $\zeta = N_m^2 = 0$, formula (4.3) becomes the corresponding formula of the gasdynamics case [3]. Formulas (4.2) and (4.3) taken together yield a quadratic equation in $S'(0)$. In the general case this can be written:

$$[a_1 + a_2\varepsilon + O(\varepsilon^2)]\, S'^2(0) + [b_1 + b_2\varepsilon + O(\varepsilon^2)]\, S'(0) + c = 0. \quad (4.4)$$

The coefficients a_1, a_2, b_1, b_2, c are found from (4.2) and (4.3). It is not difficult to derive from (4.4) a formula analogous to (4.1)

$$S'(0, \varepsilon, S_w, P) = S'(0, 0, S_w, P) + \\ + \frac{\varepsilon}{2a_1}\left[-b_2 + \frac{b_2 - 2a_2c}{b_1^2 - 4a_1c} + \right. \\ \left. + \frac{a_2}{a_1} S'(0, 0, S_w, P)\right] + O(\varepsilon^2). \qquad (4.5)$$

We shall not write the coefficients in (4.5) because of their cumbersomeness. In particular, for plane flow (n = 1), we obtain

$$S'(0) = 0.53\,(0.716\varepsilon + 1) \text{ for } S_w = 0,\ P = 1. \qquad (4.6)$$

It is interesting to compare formula (4.6) derived for the velocity profile at the outer edge of the boundary layer $U_\infty = cx^m$, $m = 1$, with the analogous formula from [1] for $m = 1/3$

$$S'(0) = 0.49\,(0.73\varepsilon + 1).$$

The effect of the flow of molten material on the value of the heat transfer at the stagnation point is about the same in both cases.

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17 May 1965 Leningrad